

PROBABILISTIC ENGINEERING ANALYSIS USING THE NESSUS SOFTWARE

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Abstract

The ability to quantify the uncertainty of complex engineering structures subject to inherent randomness in loading, material properties, and geometric parameters is becoming increasingly important in the design and analysis of structures. Probabilistic analysis provides a means to quantify the reliability of complex systems in such areas as aerospace and automotive industries. Since structural analysis predictions are often based on the results of commercial finite element (FEM) programs (e.g., ABAQUS#, ANSYS†, and MSC/NASTRAN‡), probabilistic analysis methods must be linked to such programs to achieve useful reliability results. The NESSUS§§ probabilistic analysis software combines state-of-the-art probabilistic analysis algorithms with general-purpose analysis packages to compute the probabilistic response and the reliability of engineering structures. In this paper, the NESSUS capabilities are presented and demonstrated for several application problems.

Introduction and Background

Numerical simulation is now routinely used to predict the behavior and response of complex systems. Computational simulation is being increasingly used as performance requirements for engineering structures increase and as a means of reducing testing. Since structural performance is directly affected by uncertainties associated with models or in physical parameters and loadings, the development and

application of probabilistic analysis methods suitable for use with complex numerical models is needed.

The traditional method of probabilistic analysis is Monte Carlo simulation. This approach generally requires a large number of simulations to calculate low or high probabilities, and is impractical when each simulation involves extensive finite element computations. Approximate fast probability integration (FPI) methods have been shown to be many times more efficient than Monte Carlo simulation^{1,2} and can often provide sufficient accuracy for engineering predictions. In many situations, the advanced mean value (AMV) procedure, based on FPI, can predict the probabilistic response of complex structures with relatively few response calculations. These methods also provide probabilistic sensitivity measures indicating the input parameters that influence the reliability the most.

Beginning with the development of the NESSUS probabilistic analysis computer program, Southwest Research Institute (SwRI) has been addressing the need for efficient probabilistic analysis methods for over fifteen years. NESSUS can be used to simulate uncertainties in loads, geometry, material behavior, and other user-defined random variables to predict the probabilistic response, reliability and probabilistic sensitivity measures³ of systems. NESSUS provides a built-in finite element structural modeling capability as well as interfaces to many commercially available finite element programs. This paper discusses the current capabilities of the NESSUS software and presents several application problems to demonstrate its effectiveness.

Overview of NESSUS

NESSUS is a general-purpose tool for computing the probabilistic response or reliability of engineering systems. It was initially developed by a team led by SwRI for the National Aeronautics and Space Administration (NASA) to assess uncertainties in

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critical space shuttle main engine components.⁴ The framework of NESSUS allows the user to link traditional and advanced probabilistic algorithms with analytical equations, external computer programs including commercial finite element codes, and general combinations of the two. Eleven probabilistic algorithms are available including traditional methods such as Monte Carlo simulation and the first order reliability method (FORM) and advanced methods such as the advanced mean value method and adaptive importance sampling. In addition NESSUS allows linking of different analysis packages or analytical functions to allow a general relationship of physical processes to predict the uncertainty in the performance of the system. A summary of the capabilities is shown below. The following sections describe the major components and capabilities of the NESSUS software.

Component Probabilistic and Reliability Analysis

NESSUS can compute the cumulative distribution function (CDF) of system performance (e.g. stress or strain) or include the strength of the system to compute the reliability. Traditional reliability analysis involves

computing the probability of the stress (S) exceeding the strength (R) or $P[R < S]$ or $P[g < 0]$ where

$$g = R - S . \quad (1)$$

R and S may be complex models involving other random variables such that $R(X_i)$ and $S(Y_i)$. In NESSUS, the g-function is formulated such that $g=g(X_i, Y_i)$ and thus correctly accounts for possible correlation between the stress and strength parts of the performance measure (i.e., common random variables in R and S). Capabilities in NESSUS provide a general formulation of the g-function that allows linking of different analysis codes and analytical functions. For example, a stress or strain from a finite element analysis can be used with a fatigue life equation or S-N curve to define the performance of a structure. The results for a component analysis in NESSUS are reliability or cumulative distribution function and importance factors. The importance factors indicate which variables contribute most to the unreliability and may be used for design changes or manufacturing processes modifications. Figure 1 shows a schematic of the NESSUS component analysis capability.

NESSUS 2.4 *Probabilistic Analysis Software*

Inputs

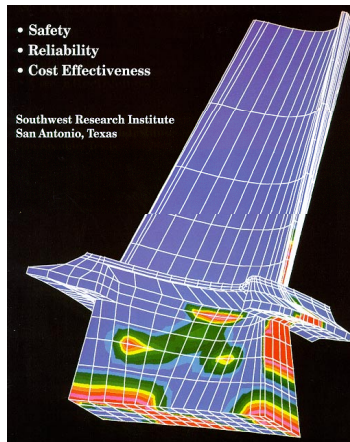
- Free format keyword interface
- Windows graphical interface (PC/FPI)
- Ten probability density functions
- Correlated random variables
- 800 page Users/Theory/Examples manual

Outputs

- Cumulative distribution function
- Prob. of failure given performance
- Performance given prob. of failure
- Probabilistic sensitivities with respect to mean and standard deviation
- Confidence bounds
- *Empirical cdf and histogram*

Analysis Methods

- First-order reliability method
- Second-order reliability method
- Fast probability integration
- Mean-value and advanced mean value
- Response surface method
- Automatic Monte carlo simulation
- Sphere-based importance sampling
- Latin hypercube simulation
- Adaptive importance sampling
- Probabilistic fault-tree
- *PFEM Simulation (MCS & LHS)*



Probability of Exceeding 80ksi

Probabilistic Analysis

- *Automated restart*
- Component reliability
- System reliability
- Reliability-based optimization
- Reliability testing plan
- Inspection scheduling
- Design certification
- Risk-based cost analysis

Performance Functions

- Analytical (Fortran)
- Analytical (Input deck)
- Numerical (FEM, BEM, other)
- Failure models (Fortran)
- Any combination of the above

NESSUS/FEM module

- Static
- Dynamics
- Nonlinear material
- 1, 2, 3D elements
- Perturbation algorithm

Interfaces

- ABAQUS
- NASTRAN
- NESSUS/FEM
- PRONTO
- User-defined

Hardware

- Personal computers
- Unix workstations
- Mainframes, supercomputers
- Y2K Compliant

Italic: new features in 2.4

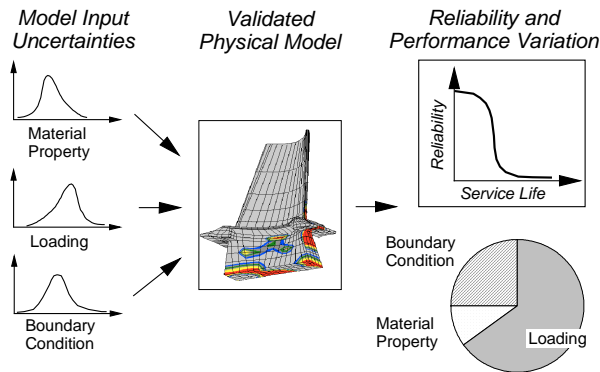


Figure 1 Component Probabilistic and Reliability Analysis in NESSUS

System Reliability Analysis

Most engineering structures can fail by multiple events including multiple failure modes and/or components in which the nonperformance of one or a combination of events can lead to system failure. System reliability considers failure of multiple components of a system and/or multiple failure modes of a component. System reliability in NESSUS is currently addressed using a probabilistic fault tree analysis (PFTA) method.⁵

System failure is defined through a fault tree by defining the bottom events and their combination with “AND” and “OR” gates. Each bottom event considers a single failure (component reliability) and can be defined by a finite element model and failure function or as an analytical equation. An example of a fault tree for a three-event system is shown in Figure 2.

Several options are available for solving system reliability problems in NESSUS. First direct Monte Carlo simulation is available but may be cost prohibitive if the bottom events consist of finite element models. NESSUS will compute the probability of failure for each event using the AMV+ method and also create a polynomial failure function at the most probable point (MPP). The failure functions are then combined based on the logic of the fault tree. The system reliability is then computed using an adaptive importance sampling method. The adaptive importance sampling method can sample the fast running functions developed for each event or the actual response model. Since the PFTA method in NESSUS uses the actual failure functions for each event, the approach can accurately account for correlation between events due to common random variables.

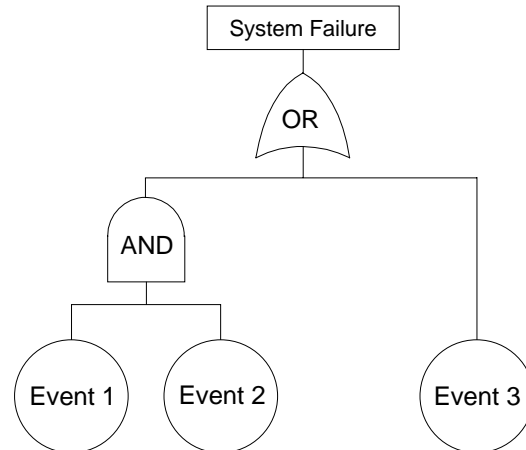


Figure 2 Example of a Fault Tree for System Reliability Analysis in NESSUS

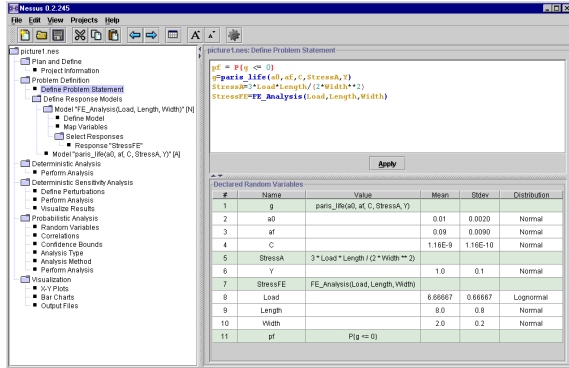
In addition to quantifying the reliability, NESSUS also computes probabilistic sensitivities of the probability of failure with respect to the random variable means and standard deviations. These sensitivities are useful in design practice to identify the variables that influence the reliability the most. The results also rank the importance of each of the events to the total probability of failure.

Graphical User Interface

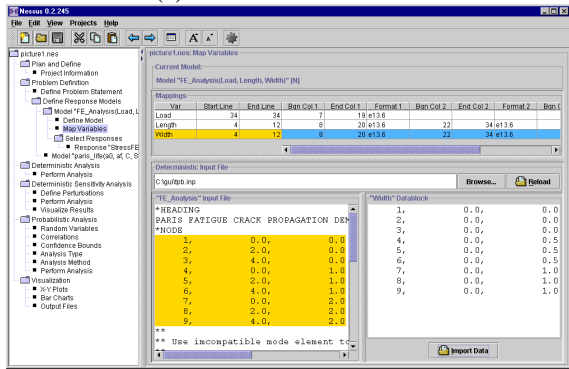
A graphical user interface (GUI) is under development to assist the user in defining the NESSUS input and postprocess the probabilistic results. Written in the Java programming language, the GUI is available on nearly all computer platforms. Several screens are shown in Figure 3 illustrating a) problem definition, b) random variable mapping, and c/d) probabilistic results.

Interfacing with External Deterministic Analysis Programs

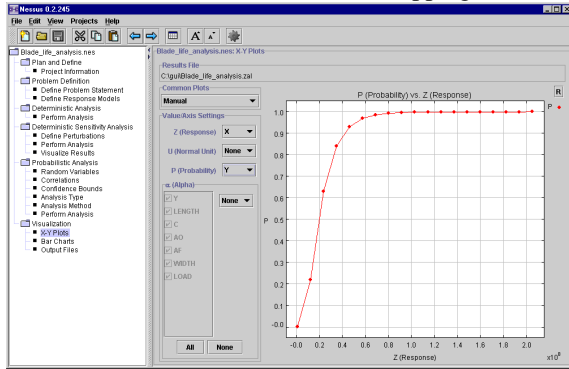
A number of issues arise when performing probabilistic analysis with external deterministic analysis programs such as commercial finite element (FE) analysis packages. These issues are described in the following sections in a general format as they apply to such analysis packages as ABAQUS, ANSYS, and NASTRAN. While the discussions focus on FE analysis programs, the approach is also applicable to other non-structural or “in-house” analysis programs.



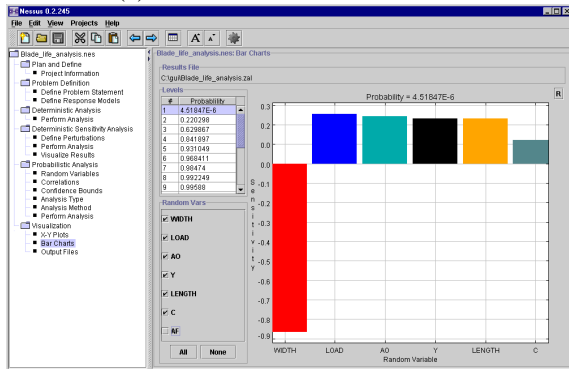
3(a) Problem statement screen.



3(b) Random variable mapping



3(c) Cumulative distribution function.



3(d) Probabilistic sensitivities.

Figure 3. Select screens from the NESSUS GUI.

The NESSUS interface to external analysis packages integrates the probabilistic analysis capabilities of the NESSUS program with the deterministic structural analysis capabilities of the required analysis. The capabilities include:

1. Any scalar quantity in analysis code input can be treated as a random variable.
2. Field variables such as geometry, pressure loading, and nodal temperatures can be modeled using fully dependent random fields.
3. All procedures in the analysis package are valid.
4. Any response variable stored in the analysis package results file can be used for the probabilistic analysis or for inclusion in a life measure model.
5. Analysis options, random variable definitions, response identification, and life measure model are all selected from within the NESSUS input file.
6. Automatic restart capability.
7. Batch mode operation.

These capabilities are described in more detail in the following section and by the example problems.

Random Variables

When performing probabilistic FE analysis, a realization of a random variable must be reflected in the FE input. A distinction is made between random variables that affect a single quantity in the FE input, called scalar variables, and random variables that affect multiple quantities, called field variables. Typical examples of scalar random variables include Young's modulus or a concentrated point load. Examples of field random variables are a pressure field acting on a set of elements or a geometric parameter that effects multiple node locations (e.g., radius of a hole).

Scalar random variables are directly mapped from the random variable value to the analysis program input. Field variables require a functional relationship between the random variable and the analysis program input. Because different realizations of these field random variables are required, a general approach is used to relate the finite element input to a change in the random variable value. For example, if the random variable is the radius of a hole, changes to a set of nodal coordinate values will be required each time the radius is changed. A shift vector, $\Delta \mathbf{x}$, is defined that relates how the coordinates change with a change in the random variable. The vector of perturbed nodal coordinates, $\hat{\mathbf{x}}$, is related to the mean value of the

coordinates, μ_x , plus a shift factor, s , times the amount of change for the coordinates, Δx , or in equation form,

$$\hat{x} = \mu_x + s \cdot \Delta x. \quad (2)$$

The shift vector is the difference between the mean value of the random variable and the perturbed value. One approach to generating Δx is to perturb the nominal mesh, subtract the nominal from the perturbed, and then normalize. This procedure, performed only once at the beginning of the analysis, is then used by NESSUS to create a finite element mesh for any value of the random variable.

Figure 4 shows an example of a field random variable, where a change in the random variable h , produces a change in the finite element mesh. This approach can be used for any type of field random variable (e.g., pressure and temperature distributions). Future versions of NESSUS will include tools for developing the required input for field random variables.

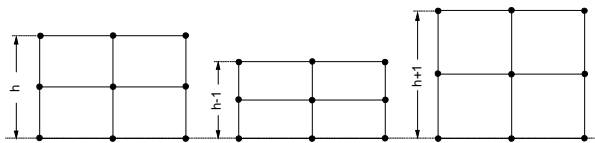


Figure 4 Example of a Field Random Variable

In some situations, inputs to the deterministic FE model are a function of the random variables defined in the NESSUS input file. For example, cross-sectional dimensions of a structural member such as width, depth, or thickness may be random variables. Rather than use these inputs directly, the deterministic FE code may alternatively require cross-sectional properties such as area and moments of inertia to be defined. To handle this and similar situations, NESSUS provides analytical relationships and a Fortran subroutine for defining FE input quantities in terms of the input random variables. Very general relationships can be defined by using an analytical function to define the relationship between FE input and the random variables.

Design Performance Measures

The performance measure for a given design is problem specific. FE analysis provides many results such as stress, strain and displacement to define performance. For nonlinear or transient problems, a measure of performance may be a plastic strain or acceleration. A key issue with combining probabilistic algorithms with FE analysis packages is extracting the results of

interest. The approach used in NESSUS is to read analysis results for a given set of node, element and time steps directly from the FE analysis package results file. General results selection options have been included in NESSUS when interfacing with FE analysis packages.

Sometimes the performance measure is a function of the FE results. For example, combining the stress or strain results with fatigue equations to develop a distribution of life. In some instances the post-processing is a closed form equation or may require calling an external program to solve for the life measure. In NESSUS, a user-defined subroutine is provided to handle both scenarios. This subroutine provides a very general mechanism to combine finite element results with a life function. Many commonly used performance models are included with NESSUS.

Probabilistic Analysis Issues

Several practical issues arise when performing probabilistic analysis for complicated models. First, the analysis may terminate for unforeseen reasons such as power outages, system maintenance or lack of disk storage space. A restart option is available in NESSUS to avoid loss of computational effort by having to start over at the beginning of the analysis should the run terminate abnormally. This restart capability allows automatic recovery from a computer crash by starting at the point where the run terminated. The restart option is supported for most analysis procedures in NESSUS. Once the FE results files have been created for a given set of random variable values, computing the probabilistic response for multiple nodes, elements or load conditions is very fast since the FE analysis is not rerun. The restart option also provides for improving a solution without rerunning any previous steps. For example, tightening the AMV+ convergence criteria or increasing the number of Monte Carlo samples would not require rerunning FE analyses associated with the previous steps of the methods.

A batch processing option is available in NESSUS to allow processing on different computers. This allows NESSUS to run on a local workstation while the finite element analysis program runs on a main frame or super computer. The batch processing option also allows for distributing the finite element analyses between different computers.

Application Examples

The NESSUS software has been used to predict the reliability and probabilistic response for a wide range of problems.^{6,7,8,9,10,11} Four problems are presented in this section to demonstrate the flexibility of the NESSUS software when linked with external analysis packages.

Reliability of an Aircraft Lever(NESSUS/ANSYS)

The example problem is a lever that is representative of a critical aircraft structural component.⁶ A deterministic analysis of a similar lever was completed for a future military aircraft using standard methods of mechanical analysis. The lever transfers load between an actuator and a control surface. The link must survive extreme, limit and normal operating loads without exceeding ultimate, yield or fatigue strengths, respectively. A deterministic analysis may assume that all geometry variables are at the weakest extremes while loads are at their highest levels. The predicted stresses associated with the different load cases must be less than A-Basis material strengths, as defined in MIL-HDBK-5. This example investigates the fatigue failure mode.

Problem Description

The finite element model of the lever including the deterministic stress contours is shown in Figure 5. The model consists of 44844 degrees of freedom and each analysis required approximate 20 minutes on an HP 700 series workstation.

The random variables for the problem are listed in Table 1. The first two random variables affect the finite element model geometry. The hole location and radius random variables are shown in Figure 5. The tolerance for each dimension is assumed to be ± 0.01 inches and chosen to represent three standard deviations from the mean.

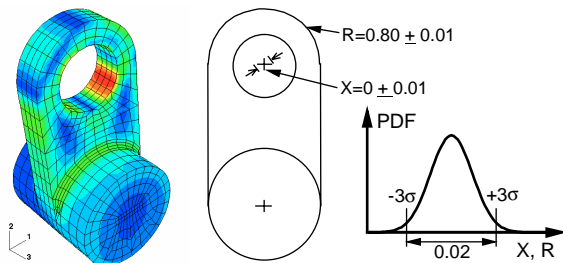


Figure 5. Finite Element Model, Stress Contours and Random Variables for Lever Example

Table 1. Random Variables for Lever Fatigue Example Problem

Name	Mean	Standard Deviation	Distribution Type
Hole Location	0.0	0.003333	Normal
Radius	0.8	0.003333	Normal
Operating Load	4.9	0.98	Normal
S-N Scatter	0	0.6	Normal

The load is assumed random and models the worst case tensile and compression loads through the range of motion of the lever. These loading extremes provide the stress range used for the fatigue life computations.

The example examines the distribution of fatigue life based on S-N data for AerMet100, shown in Figure 6. The equation for fatigue life was developed based on the data^{12,13}:

$$\text{Log}(N) = 26.4 - 9.16 \cdot \text{Log}(\sigma_{\max} (1 - R)^{0.72}) + \psi \quad (3)$$

where N is the cycles to failure, σ_{\max} is the maximum principal stress, R is the ratio of the maximum and minimum stress and ψ is the scatter factor based on regression of the S-N data.

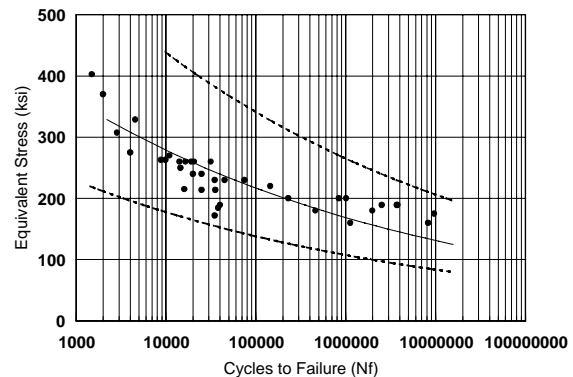


Figure 6. S-N Curve for AerMet100 with 3σ Bounds

Results

The probabilistic analysis was conducted for the fatigue life of the lever using the NESSUS interface to ABAQUS. Thirteen points of the cumulative distribution function (CDF) were computed using the AMV+ method and required 101 ABAQUS FE analyses (approximately 35 CPU hours on a 700 series HP workstation). The AMV+ CDF is shown in Figure 7 and is compared with 100 Monte Carlo samples. The Monte Carlo solution provides points on the CDF between approximately 0.01 and 0.99. For about the

same computational effort, the AMV+ solution provides a CDF range of 10^{-5} to 0.999 and covers the lower or left tail probabilities where the designer is usually most interested. The AMV+ CDF compares well with these limited Monte Carlo samples. Based on AMV+ results, the probability is 99.994% that the fatigue life of the lever will be at least 1 million cycles.

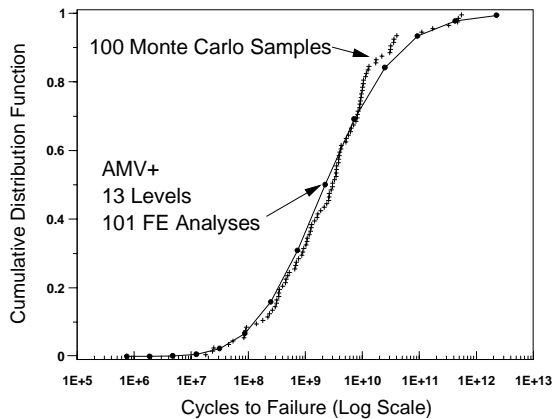


Figure 7. Cumulative Distribution Function

An important byproduct of the AMV+ method is the probabilistic sensitivity factors that identify the variables that contribute most to the reliability of the design (Figure 8). Other importance factors include the sensitivity of the p_f with respect to a change in the mean value or the standard deviation of each random variable. These normalized sensitivities are shown in Figure 9 and allow the designer to evaluate the effect of the probability of failure due to a change in a design parameter. For this example, little importance is observed on the radius and hole location and these may be reviewed for manufacturing process changes that may lead to loosening of tolerances and possible cost reduction. The importance of the scatter on the fatigue life may influence the designer to invest in improved characterization of this variable or to obtain a cleaner material.

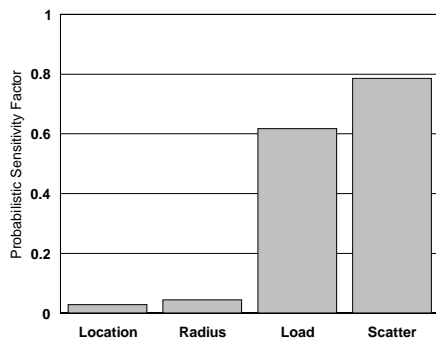


Figure 8. Probabilistic Sensitivity Factors

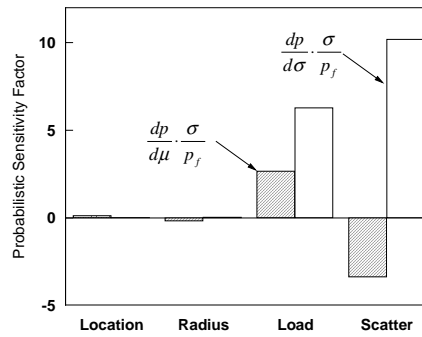


Figure 9. Sensitivity with Respect to Distribution Parameters

Probabilistic Analysis of an Advanced Fighter/Attack Aircraft Composite Wing Structure (NESSUS/NASTRAN)

An advanced lightweight composite wing structure with syncore sandwich stiffened wing skins was analyzed due to its criticality in flight performance, flight safety, and cost¹⁰. The analysis focused on the co-bonded joint that attaches the wing box lower cover to the wing spars. Variations in material properties were explored to assess the reliability of the wing at the critical wing-spar interface and to determine the important material inputs.

Problem Description

The finite element model of the wing skin is shown in Figure 10. The fuselage and the outer wing were not included in the model, but were simulated using interface loads applied to the inner wing model.

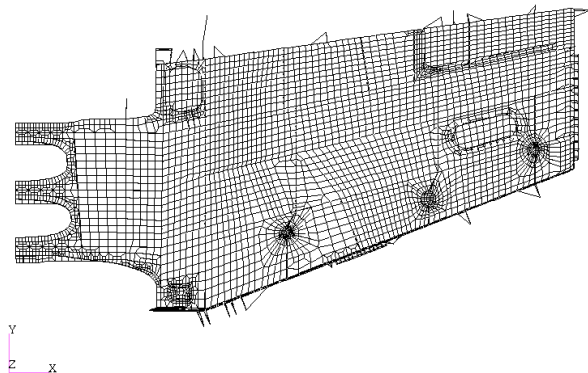


Figure 10 NASTRAN Wing Model

The critical areas of the lower wing consist of composite ply layups. Each of these layups have multiple plies varying in orientation. However, the layup orientations were not varied for this probabilistic analysis; only variations in material properties were considered.

A structural analysis model was developed that couples a global post-buckled wing analysis with a local blade analysis. NASTRAN is used for the wing analysis and BLADEM¹⁴/THELMA¹⁵ is used for the local blade analysis.

The sequence of steps in the analysis procedure is summarized in Figure 11. The steps are repeated each time NESSUS requires a deterministic solution. Failure is predicted computationally when the failure index exceeds 1.0. There are 18 independent random variables. Additional details of the analysis and a complete list of the random variables considered in this analysis can be found in the work by Millwater, et al.⁷

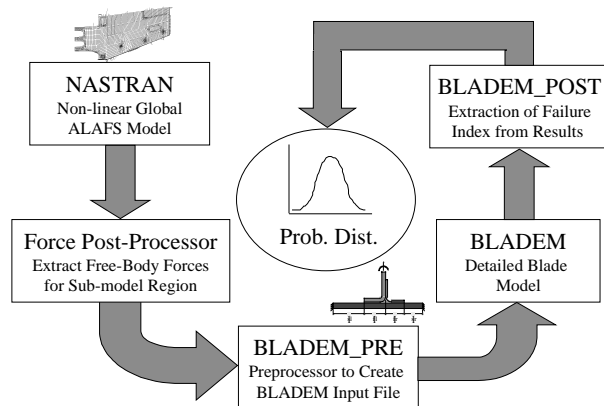


Figure 11 Wing-Blade Analysis Procedure

Results

Probability of failure calculations were performed for the major structural components of the blade (nugget, adhesive, skin plies, left and right flange plies). Table 7 below summarizes the results.

The largest probability of failure is in the first ply of the left flange at the high stressed region shown in Figure 12. This probability is denoted Ω . The probabilities of failure in the other locations of the blade are shown relative to Ω .

Table 7 Probability of Failure Results for Blade Locations

Blade Region	Probability of Failure Relative to Ω
Adhesive	0.0
Nugget	0.0
Skin Plies (1-35)	0.0
Left Flange	
Ply 1	1.0 (Ω)
Ply 2	0.53
Ply 3	0.32
Ply 4	0.22
Ply 5	0.08
Ply 6	0.05
Right Flange	
Ply 1	0.07
Ply 2	1.7E-4

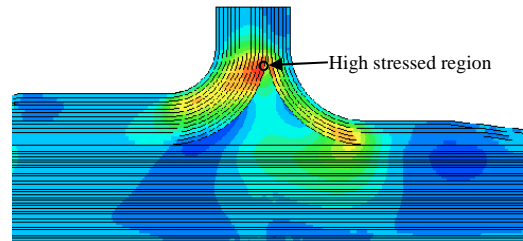


Figure 12 High Stress Region for Blade

Probabilistic sensitivities consider both the deterministic sensitivities and the scatter of the random variable. These results can be used to determine the expected changes in the probability of failure due to modifications in the mean value or standard deviation of the random variables. A complete listing of the sensitivities can be found in the reference.⁷ The results indicate that the failure index is most sensitive to the mean value and standard deviation of the cloth tensile strength and somewhat sensitive to the Cloth E2 variable.

Redesign

The analysis of the wing box-blade structure revealed that the probability of failure that was unacceptably high. Quantifying this probability of failure through probabilistic analysis also revealed a needed redesign region, and identified the random variables to modify that most cost-effectively achieved a safer structure. In this case, two geometric and one material property change was sufficient to reduce the probability of failure by many orders of magnitude such that the probability of failure of the redesigned structure is very low.

Crankshaft Reliability

The crankshaft in an internal combustion engine is subjected to hundreds of millions of cycles of loading in its lifetime. High cycle fatigue under bending and torsional loading is a common failure mode in a crankshaft. The reliability of the crankshaft is predicted using NESSUS linked with the ANSYS finite element analysis program. This work is a continuation of a crankshaft reliability analysis performed by Shah et al.⁸

Problem Description

The model is a three-dimensional, parametric model of a single crank throw. Key geometric features including the crank fillet radius and pin fillet radius and the loading are parameterized in the ANSYS model. The finite element model is shown in Figure 13.

Fatigue cracks usually initiate near the crank fillet region and propagate across the web under the crankshaft bending and torsional loading. A stress life approach based on a Goodman diagram is used to develop the failure criteria and compute the fatigue margin. Failure is assumed when the fatigue margin is less than 1.0. The loading on the crankshaft during its entire cycle was evaluated and the two crank angles that gave the worst fatigue loading were chosen for further analysis. The appropriate bending and torsional loads on the crankshaft are applied for these two load conditions and are considered as random variables (connecting rod force 1, 2 and torque). The mean and alternating stresses are computed from these two load steps and the Goodman diagram is used to evaluate the fatigue margin. The fatigue margin using the mean and alternating stress is computed directly in ANSYS. The random variables include the key geometric variables, loading, and fatigue properties as listed in Table 2.

Results

The advanced mean value method is used to solve for the reliability and compute probabilistic sensitivity information. The probability of failure was computed to be 4% (the probability that the fatigue margin is less than 1.0). The probabilistic sensitivity factors are shown in Figure 14. From the figure, the torque and the endurance limit contribute most to the probability of failure. It is also clear that the crank and fillet radii have little influence on the probability of failure. These two parameters may be considered for manufacturing process changes by loosening tolerances to potentially cut costs.

Table 2 Random Variables for Crankshaft Reliability Analysis

Name	Mean	COV	Distribution
Crank fillet Radius (Rc)	13 mm	0.5%	Normal
Pin Fillet Radius (Rp)	11.5	0.5%	Normal
Connecting Rod Force 1 (F1)	802 kN	2%	Normal
Connecting Rod Force 2 (F2)	11.8 kN	2%	Normal
Torque (T)	90. kN-m	30%	Normal
Endurance Strength (SIGe)	300 MPa	10%	Lognormal
Ultimate Strength (SIGu)	1400 MPa	10%	Lognormal

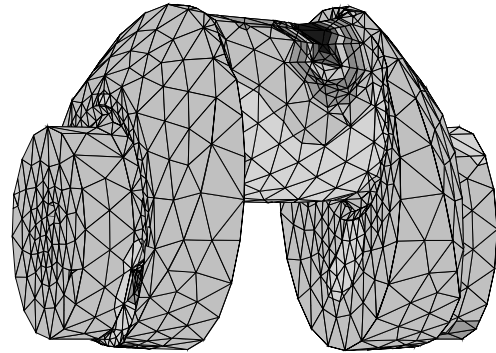


Figure 13 ANSYS Finite Element of a Crankshaft

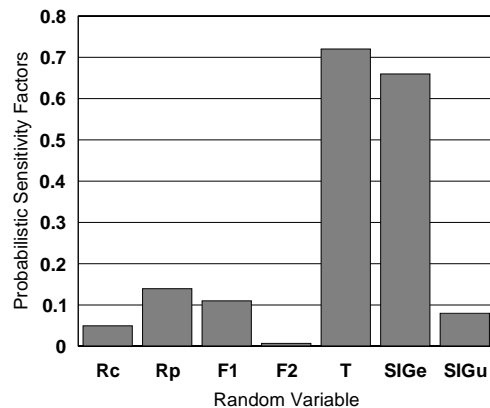


Figure 14 Probabilistic Sensitivity Factors for the Crankshaft Analysis.

Tunnel Vulnerability Assessment

The probabilistic tunnel vulnerability assessment (PVTA) involves a tunnel built into a sloped, multi-layered geology as shown in Figure 15. The tunnel vulnerability is measured as the probability that the tunnel will no longer be functional either by blocking ingress/egress or damaging equipment by airblast or fragmentation. The problem is solved by linking NESSUS with several analysis programs including the PRONTO explicit dynamic finite element program. Additional details about this analysis can be found in the work by Thacker et al.⁹

Problem Description

Two scenarios are shown in the Figure 15: one in which the weapon penetrates into the tunnel and another in which the weapon stops in the geology. Model inputs are listed in Table 3

The vulnerability of the tunnel is defined as the total probability of kill (P_k) considering all potential failure modes and their respective interactions. For the system analysis, a probabilistic fault tree (PFTA) is used to define the linkage between the bottom events through “or” and “and” gates. The PFTA method employs actual tunnel response models for each bottom event; thus, correlations between failure modes due to common random variables are fully accounted for.

Table 3 PVTA Model Inputs

Random Variable	Identifier	Mean Value	COV	Distribution
Layer 1 Thickness	t1	0.5 m	25 %	Lognormal
Layer 1 S-number	s1	10	20 %	Lognormal
Layer 2 Thickness	t2	1.0 m	25 %	Lognormal
Layer 2 S-number	s2	6	20 %	Lognormal
Layer 3 Thickness	t3	2.0 m	25 %	Lognormal
Layer 3 S-number	s3	2	20 %	Lognormal
Layer 4 Thickness	t4	5.0 m	25 %	Lognormal
Layer 4 S-number	s4	1.5	20 %	Lognormal
Layer 5 Thickness	t5	7.0 m	25 %	Lognormal
Layer 5 S-number	s5	1	20 %	Lognormal
Layer 6 S-number	s6	0.8	20 %	Lognormal
Slope Angle	θ	33.81 degrees	21 %	Lognormal
Weapon Initial Velocity	Vp	335 m/s	10 %	Lognormal
Modeling Error	B	1.0	10 %	Normal
Miss in X-direction	XMISS	0	($\sigma = 2.37$ m)	Normal
Miss in Y-direction	YMISS	0	($\sigma = 2.23$ m)	Normal
Aimpoint	X	Variable	0 %	
Retaining Wall Height	h	3 m	0 %	
Tunnel Radius	R	2 m	0 %	
Weapon Mass	Mp	8.9 kN	0 %	
Damage Function Curve-fit Coeffs.	Ci	Variable	0 %	
Rock fracture stress	TCUT	20 MPa	10 %	Normal
Equiv. plastic volume failure strain	EPVFS	-0.05%	10 %	Normal
Weapon yield	Vscale	1.0	10 %	Normal

The probabilistic fault tree employed in the tunnel vulnerability assessment is shown in Figure 16. Note that each of the three failure modes is separated into distinct target response and weapons effects bottom events.

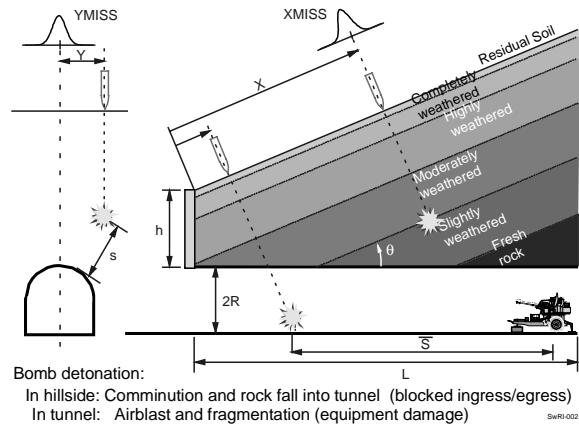


Figure 15 Conceptual Model for the Probabilistic Tunnel Vulnerability Analysis.

The NESSUS code integrated with the damage models was used to compute P_k for the three failure modes separately as shown in Figure 17(a). The comminution failure probability increases as the aimpoint is increased until hitting a maximum at $X=8$ m, at which point P_k begins to decrease. The airblast and fragmentation failure probabilities decrease monotonically as the aimpoint is increased. The optimum aimpoint for these two failure mechanisms is at $X=0$ m.

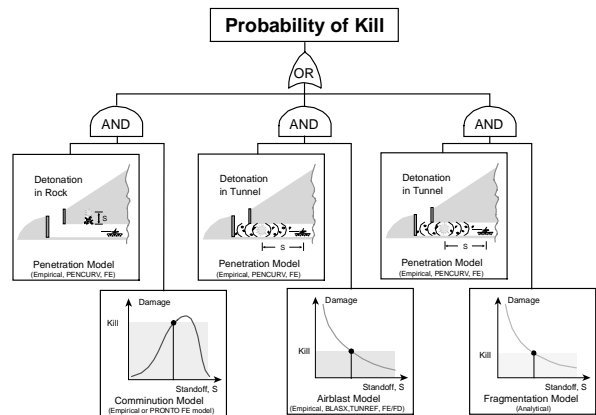


Figure 16 Probabilistic Fault Tree for the Tunnel Vulnerability Analysis for Three Failure Modes.

Results

The system P_k computed using the NESSUS PFTA method is shown in Figure 17(b). The curve labeled “assumed independent” is obtained by assuming the component failure probabilities in Figure 17(a) are statistically independent, which does not account for common random variables between events. The other curve shows the results from the PFTA, in which the

correlations between the individual failure modes are properly accounted for. The independent event assumption predicts the optimum aimpoint to be at $X=0$ m rather than at $X=8$ m when correlations are accounted for. Thus, assuming independence can lead to significant error depending on the degree of correlation between failure modes.

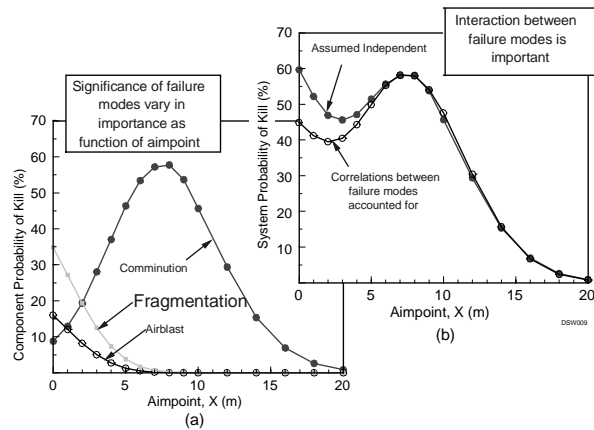


Figure 17 P_k versus Aimpoint for the Three Failure Modes: (a) Considered Separately, and (b) Combined.

Another useful product of the NESSUS PFTA analysis is the identification of which failure modes and input parameters contribute most and least to the system P_k . The sensitivity of P_k with respect to changes in the mean and standard deviation of the input parameters are shown in Figure 18 (a) and (b), respectively. From the figure, the mean value of the tensile strength (TCUT) along with several other geologic parameters contribute most to the P_k . From Figure 18 (a) the standard deviation of the penetrator initial velocity as well as several geologic parameters are important.

The analysis shows flexibility of the NESSUS software for performing complex system analysis when a diverse set of deterministic analysis codes are required to define system performance. The analysis also clearly shows that correlations between failure modes must be accounted for to accurately compute the performance of the system.

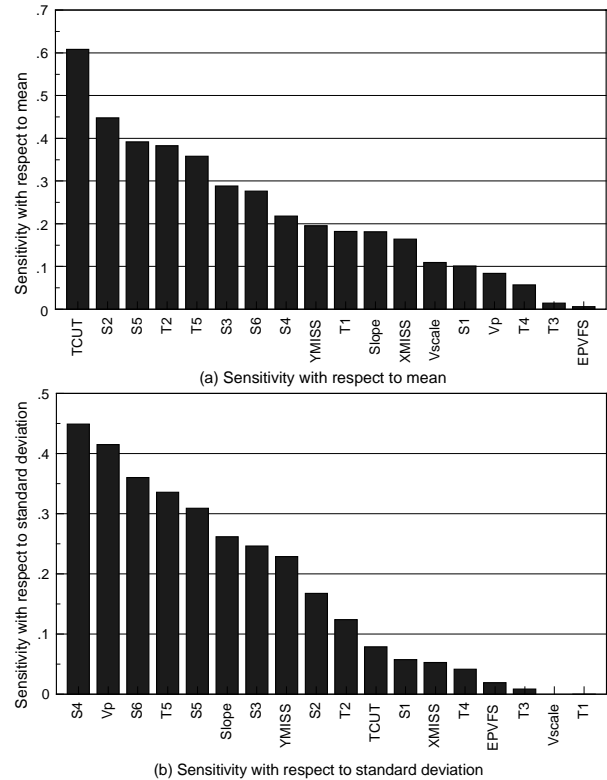


Figure 18 Probabilistic Sensitivity Measures: (a) sensitivities with respect to changes in the mean value, (b) sensitivities with respect to change in the standard deviation.

Conclusions

The probabilistic methods available in NESSUS were initially developed for aerospace applications; however, the methods are broadly applicable and their use warranted in situations where uncertainty is known or believed to have a significant impact on the structural response. The framework of NESSUS allows the user to link traditional and advanced probabilistic algorithms with analytical equations, commercial finite element programs and “in-house” analysis packages to generate the probabilistic response or reliability of a system.

The probabilistic algorithms in NESSUS include both traditional methods such as Monte Carlo simulation and the first order reliability method (FORM) and advanced methods including the advanced mean value method and adaptive importance sampling. In addition NESSUS allows linking of different analysis packages or analytical functions to allow a general relationship of physical processes to predict the uncertainty in the performance of the system.

To aid acceptance of using probabilistic methods in design, the probabilistic algorithms must be interfaced with widely used commercial analysis packages such as ABAQUS, ANSYS, and NASTRAN, and be easy to use. This integration with commercially-available software allows the use of the best tool for the problem, one in which the designer is confident in using, or a tool that an organization has invested in for training or development. The development of a platform-independent graphical user interface for NESSUS makes the probabilistic analysis straightforward to setup and understand.

Several probabilistic analyses were presented that demonstrated the flexibility of the NESSUS software when linked with deterministic analysis packages such as ANSYS, ABAQUS and NASTRAN. The advanced probabilistic analysis methods in NESSUS allow for using high-fidelity models to define the structure or system even when each deterministic analysis may take several hours to run. In the application problems presented, the probabilistic results revealed additional information that would not have been available if traditional deterministic approaches were used.

Progress in probabilistic mechanics relies strongly on the development of validated deterministic models, systematic data collection and synthesis to resolve probabilistic inputs, and identification and classification of failure modes. Future work in this area should include a more efficient means of computing perturbed finite element solutions. One approach would be a tighter link to commercial finite element analysis packages that have design sensitivity capabilities such as ABAQUS, NASTRAN and Pro/MECHANICA. This would require the commercial software developers to integrate the probabilistic capabilities of NESSUS directly into their software or provide links for more efficient communication with NESSUS.

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