

PROBABILISTIC FINITE ELEMENT ANALYSIS OF THE HUMAN LOWER CERVICAL SPINE

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ABSTRACT

The probabilistic response of an anatomically accurate, three-dimensional, non-linear and experimentally validated finite element model of the human lower cervical spine is presented. Biological variability of the model input variables is accounted for by modeling these parameters as random variables. Efficient probabilistic methods are used to determine the effect of biological variability on the computed response of the non-linear finite element model. Probabilistic sensitivity factors are determined to assess the relative importance of each of the random variables on the probabilistic response.

1. INTRODUCTION

Cervical spine injuries occur as a result of impact or from large inertial forces such as those experienced by military pilots during ejections, carrier landings, and ditchings. Other examples include motor vehicle, diving, and athletic related accidents. Reducing the likelihood of injury by identifying and understanding the primary injury mechanisms and the important factors leading to injury motivates most research in this area.

Because of the severity associated with most cervical spine injuries, it is of great interest to design occupant safety systems that minimize the probability of injury. To do this, the designer must have quantified knowledge of the probability of injury due to different impact scenarios, and also know which model parameters contribute the most to the injury probability. Although complex numerical (e.g., finite element) models are becoming more widely used as a means of augmenting and extending laboratory testing, most are deterministic in that they do not quantify the effect of uncertainties on the computed model responses.

Stress analysis plays a critical role in understanding the mechanics of injury and the effects of degeneration as a result of disease on the structural performance of spinal segments and finite element analysis (FEA) is the method of choice to conduct these analyses. The advantages of using FEA are clear: complex geometry and boundary conditions can be modeled; inhomogeneous and non-linear materials simulated, parametric studies isolating the effect of one or more variables can be performed, and delineating the stress distributions in the various components of the biological structure can be accomplished. This information may serve as a basis to evaluate the response of the normal structure and the effects of degeneration or surgical interventions.

In many structural systems, however, there is a great deal of uncertainty associated with the environment in which the structure is required to function; this variability or uncertainty has a direct effect on the structural response of the system [1]. Biological systems are an archetype example: uncertainty and variability exist in the physical and mechanical properties and geometry of the bone, ligaments, cartilage, as well as uncertainty in joint and muscle loads.

An extensive amount of experimental work has been conducted using isolated vertebra, cervical spinal columns, and cadavers to better understand the mechanisms of cervical spine injury, and to measure material properties for use in developing and validating numerical models. Static experiments have been conducted by many researchers [2-4] for purposes of understanding the basic kinematics of spine motion and for use in verifying numerical models. Many test programs have measured the response of the cervical spine under dynamic loading as well [2,5-8]. Unfortunately, nearly all static or dynamic experiments concentrate on the measurement of the gross overall motion rather than detailed stresses and strains, which are generally more useful for validating numerical models. Some experimental work has been performed to assess the role of ligaments on the stability of the spine [9] and to determine the paths of predominate load transmission through the vertebra and discs [10]. Tests aimed at determining the fracture strength of vertebral bone [11], compressive strength of discs, and tensile strength of ligaments [7] have also been performed.

Latham [12] is usually cited as the first to develop a mathematical model for describing the dynamic response of the spine to vertical acceleration. Continuum models employing three-dimensional (solid) elements are now beginning to emerge due to the availability of improved computational resources. The first continuum model was proposed by Hess and Lombard [13], which included only axial response. Subsequently, Moffat, et al., [14] included bending effects using a beam type model. Liu, et al., [15] developed bar-beam models that accounted for large displacements in the analysis. Kleinberger [16] developed a human cervical spine model that included cervical vertebrae, intervertebral discs, and biomechanically relevant spinal ligaments. More recently, Voo, et al., [17] describes the development of a finite element model of C4-C6 using an integration of CT scans, solid modeling, and automatic mesh generation. Reviews of mathematical and finite element modeling of biomaterials are given by Yoganandan, et al., [18,19].

Under the action of general multi-axial loading, multiple injury mechanisms will generally exist. To develop and validate numerical models, an understanding of the mechanisms of injury is required. Kazarian [20], Sances, et al., [21], Myers, et al., [22], and White and Panjabi [23] provide comprehensive discussions on cervical spine injury mechanisms. Stable injuries are those in which there is no danger of spinal cord damage or injury to nerve roots, whereas unstable injuries are those in which there is the likelihood of trauma to the spinal cord or nerve roots. Stability depends on the fracture configuration and integrity of the ligaments.

The broad objective of this investigation is to explore how uncertainties influence the performance of an anatomically accurate, three-dimensional, non-linear, experimentally validated finite element model of the human lower cervical spine.

2. METHODS

Efficient probabilistic methods were used to calculate the probabilistic response of the cervical spine finite element model [24,25]. These methods have been primarily developed for complex computational systems requiring time-consuming calculations, the results of which have been shown to approach the exact solution obtained from traditional Monte Carlo methods using significantly fewer function evaluations [26].

2.1 Probabilistic Injury Function

The cervical spine injury function is written as

$$Z(\mathbf{X}) = T - R(\mathbf{X}) \quad (1)$$

where $Z(\mathbf{X})$ is the injury function, T is the injury tolerance, $R(\mathbf{X})$ is the spine response due to impact or inertial forces, and \mathbf{X} is a vector of n random variables. If T is also considered to be a random variable, then the random variable vector \mathbf{X} will be assumed to include T when referencing $Z(\mathbf{X})$ in the following discussions. From Equation 1, the limit-state (or g -) function can be defined as

$$g = Z(\mathbf{X}) - z_0 \quad (2)$$

where z_0 is a particular value of $Z(\mathbf{X})$. As a simple example, T could be the yield stress of the vertebral cancellous bone and $R(\mathbf{X})$ could be the von Mises stress computed in the bone from a finite element analysis for a particular \mathbf{X} . In the usual case, $z_0 = 0$ and injury is assumed to occur when $T \leq R(\mathbf{X})$, i.e., $g \leq 0$. The limit-state given by Equation 2 can also easily be written for other injury criteria such as bone fracture or disc rupture.

The random variables \mathbf{X} represent the cervical spine model variables, such as loading (e.g., magnitude and rate of onset), initial conditions (e.g., spine position, initial velocity), boundary conditions (e.g., helmet, restraining system), material properties, and geometry. Each random variable is defined by a probability density function (p.d.f.), which is usually defined by a few parameters such as a mean value, standard deviation and distribution type. To develop the p.d.f.'s, repeat test data are needed for each parameter. Alternatively, reasonable assumptions can be made based on data available in the literature, clinical data, and/or expert opinion. To handle situations where no data is available, the sensitivities of the probability of injury with respect to the input distribution parameters can be computed and studied to identify critical modeling areas and data-collection needs.

The condition $g = 0$ defines the limit-state surface, which separates the variable space into injury $g \leq 0$ and non-injury $g > 0$ regions. Probability of injury can then be defined as

$$p_I = \Pr[Z(\mathbf{X}) \leq z_0] = \Pr[g \leq 0] = \int_{\Omega} \dots \int f_x(\mathbf{X}) d\mathbf{X} \quad (3)$$

where $f_x(\mathbf{X})$ is the joint p.d.f. of \mathbf{X} and Ω is the failure region defined where $g(\mathbf{X}) \leq 0$. The p_I corresponding to $z_0 = 0$ defines one point in the cumulative distribution function (c.d.f.); the complete c.d.f. can be constructed by repeating the calculation of p_I for many different z_0 values. The joint p.d.f. in terms of iso-probability contours, limit-state, and failure region is illustrated in Figure 2 for the case of two random variables.

Except for a few trivial cases, Equation 3 requires numerical solution such as Monte Carlo simulation. However, if Equation 2 involves a complicated finite element model and/or if p_I is relatively small, Monte Carlo simulation can become impractical due to the large number of simulations needed to reduce sampling error to an acceptable level. Therefore, it is often desirable to use approximate methods, such as most probable point methods, that are more efficient than Monte Carlo simulation. Several such methods are implemented in the NESSUS computer program [27] and are suitable for this application. NESSUS has a built-in finite element structural modeling capability and interfaces to many other commercial finite element programs, and can simulate uncertainties in the loads, geometries, material behavior, and other user-defined uncertainty inputs. NESSUS was initially developed by a team led by Southwest Research Institute (SwRI) for the National Aeronautics and Space Administration (NASA) to assess uncertainties in critical space shuttle main engine components [28-31].

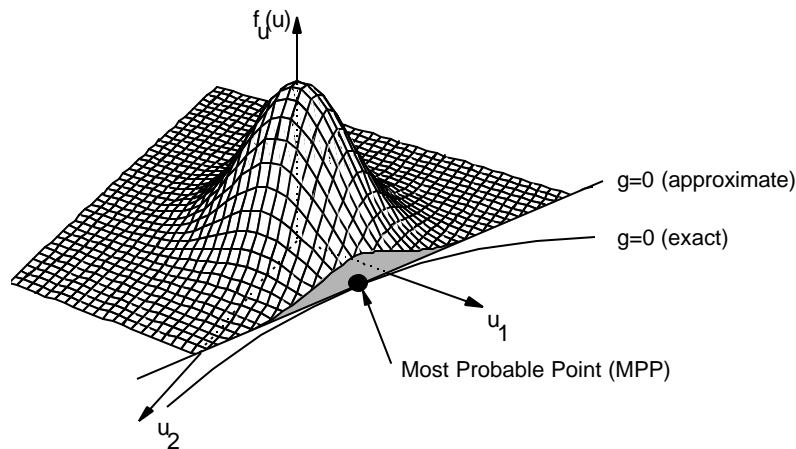


Figure 1. Joint probabilistic density function in standard normal space showing exact and approximate g-function and most probable point.

2.2 Most Probable Point Methods

A class of probabilistic methods based on the most probable point (MPP) are becoming routinely used as a means of reducing the number of g -function evaluations from that of brute-force Monte Carlo simulation. Although many variations have been proposed, the best-known and most widely-used MPP-based methods include the first-order reliability method (FORM) [32], second-order reliability method (SORM) [33], and advanced mean value (AMV) [26].

The basic steps involved in MPP-based methods are as follows: (1) Obtain an approximate fit to the exact g -function at \mathbf{X}^* , where \mathbf{X}^* is initially the mean random variable values; (2) Transform the original, non-normal random variables into independent, normal random variables \mathbf{u} [32]; (3) Calculate the minimum distance, β (or safety index), from the origin of the joint p.d.f. to the limit state surface, $g = 0$. This point, \mathbf{u}^* , on the limit state surface is the most probable point (MPP) in the u -space; (4) Approximate the g -function $g(\mathbf{u})$ at \mathbf{u}^* using a first or second-order polynomial function; and (5) Solve the resulting problem using available analytical solutions [1].

Step (1), which involves evaluating the g -function, represents the main computational burden in the above steps. Once a polynomial expression for the g -function is established, it is a numerically simple task to compute the failure probability and associated MPP. Because of this, the complete response c.d.f. can be computed very quickly by repeating steps (2)-(4) for different z_0 values. The resulting locus of MPP's is efficiently used in the advanced mean value algorithm (discussed next) to improve the probability estimates in the tail regions.

2.3 Advanced Mean Value Method

The advanced mean value class of methods are most suitable for complicated but well-behaved response functions requiring computationally-intensive calculations. Assuming that the response function is smooth and a Taylor's series expansion of Z exists at the mean values, the mean value Z -function can be expressed as

$$Z_{MV} = Z(\boldsymbol{\mu}) + \sum_{i=1}^n \left. \frac{\partial Z}{\partial X_i} \right|_{\boldsymbol{\mu}} (X_i - \mu_i) + H(\mathbf{X}) \quad (6)$$

where Z_{MV} is a random variable representing the sum of the first order terms and $H(\mathbf{X})$ represents the higher-order terms.

For nonlinear response functions, the MV first-order solution obtained by using Equation 6 may not be sufficiently accurate. For simple problems, it is possible to use higher-order expansions to improve the accuracy. For example, a mean-value second-order solution can be obtained by retaining second-order terms in the series expansion. However, for problems involving implicit functions and large n , the higher-order approach becomes difficult and inefficient.

The AMV method improves upon the MV method by using a simple correction procedure to compensate for the errors introduced from the truncation of the Taylor's series. The AMV model is defined as

$$Z_{AMV} = Z_{MV} + H(Z_{MV}) \quad (7)$$

where $H(Z_{MV})$ is defined as the difference between the values of Z_{MV} and Z calculated at the Most Probable Point Locus (MPPL) of Z_{MV} , which is defined by connecting the MPP's for different z_0 values. The AMV method reduces the truncation error by replacing the higher-order terms $H(\mathbf{X})$ by a simplified function $H(Z_{MV})$. As a result of this approximation, the truncation error is not optimum; however, because the R -function correction points are usually close to the exact MPP's, the AMV solution provides a reasonably good solution.

The AMV solution can be further improved by using an updated expansion point, which is usually obtained from the MV solution. Based initially on Z_{MV} and by keeping track of the MPPL, the exact MPP for a particular limit state $Z(\mathbf{X}) - z_0$ can be computed to establish the AMV+ model, which is defined as

$$Z_{AMV+} = Z(\mathbf{X}^*) + \sum_{i=1}^n \frac{\partial Z}{\partial X_i} \Big|_{X_i^*} (X_i - \mu_i^*) + H(\mathbf{X}) \quad (8)$$

where \mathbf{X}^* is the converged MPP. The AMV-based methods have been implemented in NESSUS and validated using numerous problems [26,29,30,34].

2.4 Probabilistic Sensitivity Analysis

For design purposes, it is important to know which problem parameters are the most important and the degree to which they control the design. This can be accomplished by performing sensitivity analyses. In a deterministic analysis where each problem variable is single-valued, design sensitivities can be computed that quantify the change in the performance measure due to a change in the parameter value, i.e., $\partial Z/\partial X_i$.

As stated earlier, each random input variable is characterized by a mean value, a standard deviation, and a distribution type. That is, three parameters are defined as opposed to just one. The performance measure is the injury probability (or safety index). Sensitivity measures are needed then to reflect the relative importance of each of the probabilistic parameters on the probability of injury. NESSUS computes probabilistic based sensitivities for both MPP and sampling based methods; details are given by Wu [35] and Thacker, et al. [36, 37]. A sensitivity computed as a by-product of MPP-based methods is

$$\alpha_i = \frac{\partial \beta}{\partial u_i} \quad (9)$$

which measures the change in the safety index with respect to the standard normal variate u . Although useful for providing an importance ranking, this sensitivity is difficult to use in design because u is a function of the variable's mean, standard deviation, and distribution. Two other sensitivities that are more useful for design (and for importance ranking as well) include

$$S_\mu = \frac{\partial p_I}{\partial \mu_i} \sigma_i \quad (10)$$

which measures the change in the probability of injury with respect to the mean value; and

$$S_\sigma = \frac{\partial p_I}{\partial \sigma_i} \sigma_i \quad (11)$$

which measures the change in the probability of injury with respect to the standard deviation. Multiplying by σ_i nondimensionalizes the sensitivity to facilitate comparison between variables.

2.5 Finite element Model

An experimentally validated three-dimensional finite element model of the C4-C5-C6 spinal segment developed at the Medical College of Wisconsin [38] was used to calculate the structural response of the lower cervical spine and to quantify the effect of uncertainties on the performance of the biological system. The ABAQUS commercial finite element program [39] was used to perform the calculations. The load-deflection response was validated against experimental results from eight cadaver specimens [40]. The moment-rotation response of the finite element model was validated against experimental results reported in the literature [41].

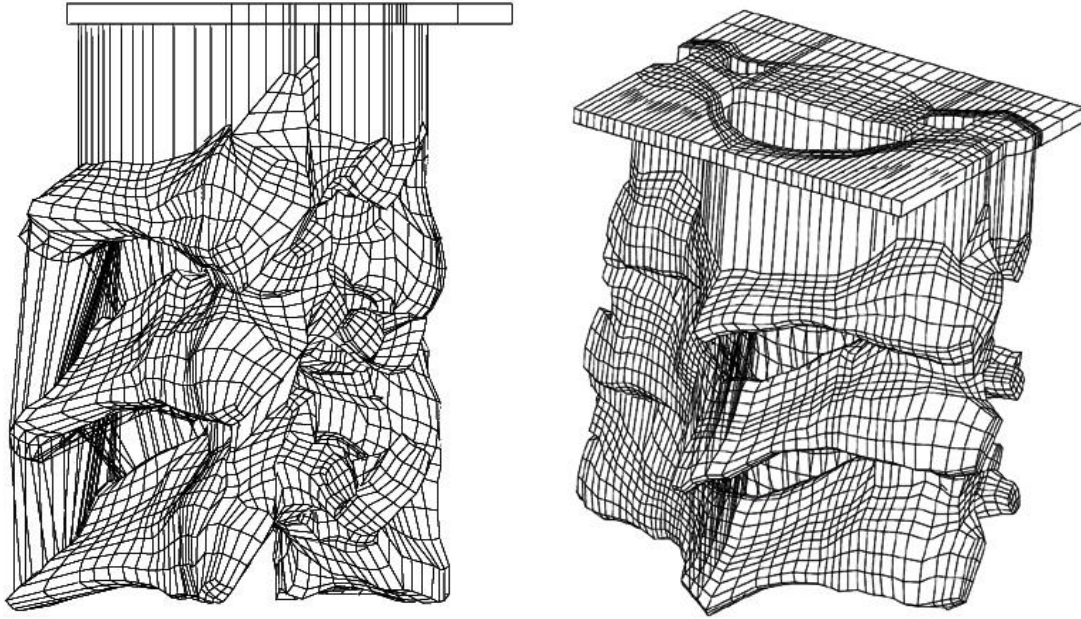


Figure 2. Finite element model of the lower cervical spine.

The probabilistic finite element model was exercised under flexion loading by applying a pure bending moment of 2N-m to the superior surface of the C4 vertebra. The inferior surface of the C6 vertebra was fixed in all directions. The resulting moment-rotation behavior was quantified by determining the rotation between the superior aspect of C4 and the fixed boundary of C6 and monitoring the reaction forces at the fixed boundary condition.

2.6 Random Variables

Biological variability was accounted for by modeling material properties and spinal segment loading as random variables. In Table 1, the mean values were set equal to the nominal values used in [38]. Thus, the original deterministic solution will be obtained when the random variables are set equal to their mean values. Since data were not available to characterize the model inputs sufficiently, it was decided to model all random variables with a coefficient of variation (cov) of ten percent. The standard deviation is computed as the mean multiplied by the cov. Finally, default distributions were assigned based on experience, i.e., a lognormal distribution was used to model modulus variables and a normal distribution was used otherwise.

A mapping must be defined to relate the random variable values to actual finite element input quantities. Variables 6-28 in Table 1 were straightforward to define because each random variable mapped directly into an ABAQUS input item (one for one). The first five variables and variable number 29 in Table 1, however, map into multiple ABAQUS inputs. This was set up by defining perturbation functions in a NESSUS user subroutine. As an example, the MALLRV random variable characterizes the uncertainty in the force-deflection relationship of the anterior longitudinal ligament. When MALLRV changes, the complete tabular force-deflection relationship defined in the ABAQUS input files changes. The mean value force-deflection relationship and the -3σ and $+3\sigma$ curves are shown in Figure 3. With these relationships defined in NESSUS, the probabilistic analysis is completely automated.

Table 1. Cervical Spine Finite Element Model Random Variables

No.	Name	Description	Mean	Standard Deviation	Distribution
1	MALLRV	Anterior longitudinal ligament nonlinear spring force-deflection relationship.	0	1.0	Normal
2	MPLLRV	Posterior longitudinal ligament nonlinear spring force-deflection relationship.	0	1.0	Normal
3	MISRV	Interspinous ligament nonlinear spring force-deflection relationship.	0	1.0	Normal
4	MCLRV	Capsular ligament nonlinear spring force-deflection relationship.	0	1.0	Normal
5	MLFRV	Ligamentum flavum nonlinear spring force-deflection relationship.	0	1.0	Normal
6	C45AREA	45 disc cross-sectional area (rebar element) inner/middle/outer & in/out.	0.195	0.0195	Lognormal
7	C56AREA	56 disc cross-sectional area (rebar element) inner/middle/outer & in/out.	0.1985	0.01985	Lognormal
8	C4PE	C4 posterior Young's modulus.	3500	735	Lognormal
9	C5PE	C5 posterior Young's modulus.	3500	735	Lognormal
10	C6PE	C6 posterior Young's modulus.	3500	735	Lognormal
11	C4CE	C4 cancellous Young's modulus.	100	30	Lognormal
12	C5CE	C5 cancellous Young's modulus.	100	30	Lognormal
13	C6CE	C6 cancellous Young's modulus.	100	30	Lognormal
14	A45E	Annulus C45 Young's modulus.	4.7	0.705	Lognormal
15	LM45E	Lusckha membrane C45 Young's modulus.	12	1.2	Lognormal
16	A56E	Annulus C56 Young's modulus.	4.7	0.705	Lognormal
17	LM56E	Lusckha membrane C56 Young's modulus.	12	1.2	Lognormal
18	SM456E	Synovial membrane C45/C56 facet Young's modulus.	12	1.2	Lognormal
19	NP45FD	Nucleus pulposus C45 fluid density.	0.000001	0.0000001	Normal
20	NP56FD	Nucleus pulposus C45 fluid density.	0.000001	0.0000001	Normal
21	FSF45FD	Facet synovial fluid C45 fluid density.	0.000001	0.0000001	Normal
22	FSF56FD	Facet synovial fluid C56 fluid density.	0.000001	0.0000001	Normal
23	LJ45FD	Luschka's joint C45 fluid density.	0.000001	0.0000001	Normal
24	LJ56FD	Luschka's joint C56 fluid density.	0.000001	0.0000001	Normal
25	CORTE	Cortical Young's modulus.	12000	2520	Lognormal
26	ENDPE	Endplate Young's modulus.	600	126	Lognormal
27	CARTE	Cartilage Young's modulus.	10.4	1.04	Lognormal
28	FIBRE	Fiber Young's modulus.	500	50	Lognormal
29	FLEXLOAD	Flexion loading.	1	0.01	Normal

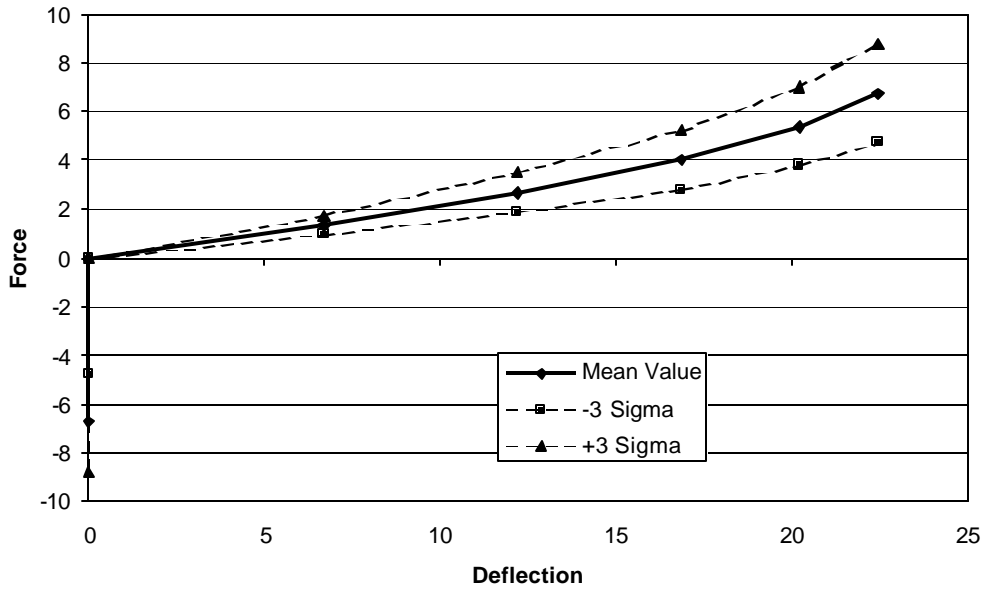


Figure 3. Mean value and perturbed anterior longitudinal ligament nonlinear spring model.

3. RESULTS

The probabilistic rotation response had an approximate mean of 3.82 degrees and a standard deviation of 0.38 degrees resulting in a coefficient of variation of 10%. The cumulative distribution function for rotation is shown in Figure 1. The c.d.f. is used to determine probabilities directly, e.g., the cumulative probability at 4.2° is 82%.

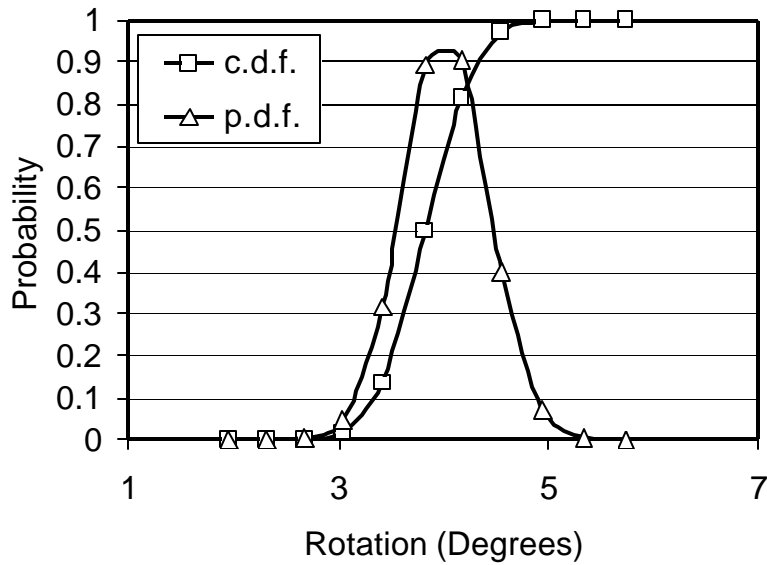


Figure 4. Cumulative distribution function and corresponding probability density of rotation.

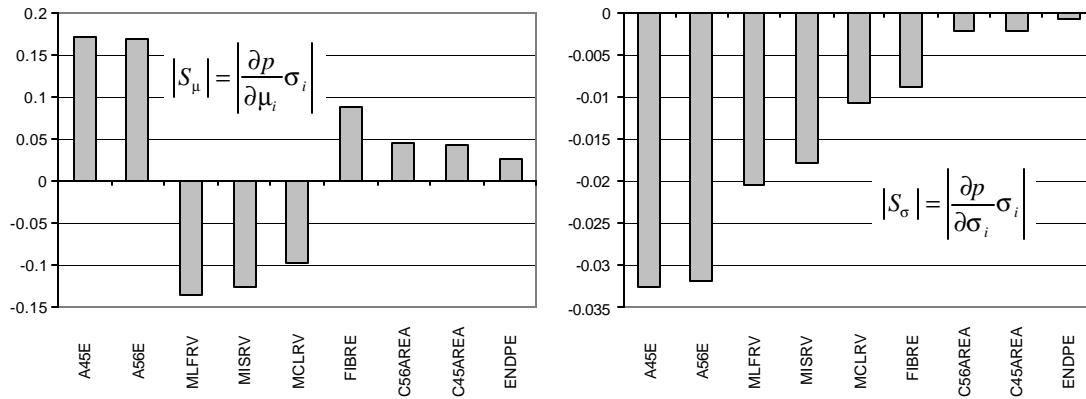


Figure 5. Probabilistic sensitivity factors.

Figure 5 shows the sensitivity information for the eight most significant random variables. The applied loading (FLEXLOAD) is the dominate variable (not shown); thus, FLEXLOAD was removed from Figure 5 so that the other variables can be more clearly seen. Not including FLEXLOAD, the most important variables are the mean values of the : 1) C45 and C56 intervertebral disk annulus Young’s modulus, 2) Ligamentum flavum nonlinear spring force-deflection relationship 3) interspinous ligament nonlinear spring force-deflection relationship, and 4) capsular ligament nonlinear spring force-deflection relationship.

The probabilistic sensitivities can be interpreted as follows: a small change in the mean value of the important variables (e.g., FLEXLOAD, disk annulus modulus) will result in change in the computed probability. Positive sensitivity values indicate a positive change in the mean value will result in an increase in the computed probability. Negative sensitivities indicate that a positive increase in the mean value will result in a decrease in the computed probability. The sensitivities S_σ indicate how a change in a random variable’s scatter (standard deviation) will change the computed probability. These results can be used to eliminate unimportant variables from the random variable vector thus improving computational efficiency. Furthermore, resources can be focused on further characterization efforts for those variables that are significant.

4. DISCUSSION

Although definitive conclusions should not be drawn from this preliminary analysis, the probabilistic results reveal some interesting characteristics regarding the spinal injury potential—findings that would not be available from conventional deterministic analysis techniques. The computed injury probability is, of course, strongly associated with what is assumed to constitute injury (in this case the injury tolerance), the nature of the uncertainty modeled in the analysis, and the physical mechanisms that relate the response of the model to the applied loading. Conclusions drawn from this analysis must bear these assumptions in mind.

The probabilistic methods used herein were initially developed for aerospace applications and are broadly applicable. Their use is warranted in situations where uncertainty is known or believed to have significant impact on the structural response. Furthermore, if an appropriate failure or injury metric is defined, the probability of failure or injury can be quantified. This emphasizes the additional information and advantage of performing a probabilistic analysis as compared to a deterministic analysis. For example, the cumulative probability at 4.2° rotation is 82%; thus, if failure is assumed when rotation exceeds 4.2° , the probability of failure is $1-0.82 = 18\%$. With the mean value analysis, the computed rotation is less than the rotational failure limit (3.8° vs. 4.2°), thus indicating a non-injurious condition. However, as noted, by performing a probabilistic analysis, the likelihood or probability of exceeding the injury threshold is quantified (18%).

In this analysis, random variable parameters were assumed. Specifically, mean values were assumed to be the deterministic (nominal) values from the validated finite element model. Standard deviations were computed by assuming a coefficient of variation of 10%, and probability distributions were assumed based on experience. Of the 29 random variables considered in the analysis, the probabilistic sensitivity information suggests that data collection efforts be focused on the loading (FLEXLOAD). To a lesser but still significant degree, the results also suggest that data be collected for A45E, A56E, MLFRV, MISRV, MCLRV, and FIBRE. Since these variables contribute the most to the computed probability, improved estimates for the mean, standard deviation, and distribution will have the most impact on the computed probability.

5. CONCLUSIONS

A probabilistic methodology and computational tool was developed to explore how uncertainties influence the performance of an anatomically accurate, three-dimensional, non-linear, experimentally validated finite element model of the human lower cervical spine. The calculations were performed using the NESSUS probabilistic analysis program integrated with the commercial ABAQUS finite element program. The probabilistic methods, which were initially developed for aerospace applications, are broadly applicable and their use warranted in situations where uncertainty is known or believed to have a significant impact on the structural response.

The probabilistic analysis methods presented herein provide a new tool with which inherent uncertainties and variations in biological structures can be studied and the impact of their role on the biomechanical response better understood. Potential applications of the methodology include the study of spinal behavior under normal and distressed conditions, the design of implants utilizing novel materials and/or configurations, analysis of novel instrumentation systems that may help avoid costly experimentation, and the design of anthropomorphic test devices or physical models that replicate human response such that injury under simulated dynamic conditions can be replicated.

Progress in probabilistic biomechanics depends critically upon development of validated deterministic models, systematic data collection and synthesis to resolve probabilistic inputs, and identification and classification of clinically-relevant injury modes. Future work in this area should include development of a high-fidelity continuum cervical spine model for assessing the combined likelihood of vertebral fracture, ligament sprain and disc rupture. Additionally, research is needed to integrate random process loading and random field representations of geometrical variations and initial configuration into the probabilistic model. To be directly useful in design, the probabilistic methodology must be integrated into an occupant safety design tool such that the sensitivity of design (controllable) parameters are related to probability of injury.

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